Ingegneria delle Telecomunicazioni

Satellite Communications

19. Cold and Warm – Ranging Code Acquisition and Tracking

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TOA Measurement: Basics of Estimation Theory

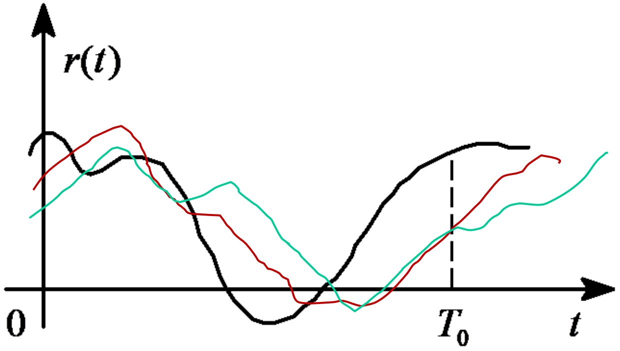
Available observation: the received waveform $r(t;\tau), \quad 0 \le t < T_0$

***Estimate** of the parameter :* $\hat{\tau} = F\left[r(t;\tau)\right]$

The estimate $\hat{\tau}$ *is **unbiased** if* $E\{\hat{\tau}\} = \tau$

*Estimator **MSE*** $MSE(\hat{\tau}) = E\left\{\left(\hat{\tau} - \tau\right)^2\right\}$

*Estimator **Variance*** $\sigma_{\hat{\tau}}^2 = E\left\{\left(\hat{\tau} - E\{\hat{\tau}\}\right)^2\right\}$



Because of noise, the estimate is a random variable; its accuracy (i.e., its MSE), depends on the amount of noise or on the SNR !

Basic Modeling of the I/Q received signal

- N_{sat} number of satellites in visibility *(elevation larger than $\approx$10 degrees)*
- *i* satellite identifier
- C_i received signal power
- τ_i time-of-flight in the user time scale (group delay)
- Δf_i carrier Doppler shift
- θ_i satellite carrier phase shift

$$r(t)=\sum_{i=1}^{N_{sat}}\sqrt{2C_i}\,s_i(t-\tau_i)\exp\left[j\left(2\pi\Delta f_i t+\theta_i\right)\right]+w(t)$$

Let us super-simplify and consider just the delay from 1 satellite as the only parameter

The (Modified) Cramér-Rao Bound

*The (**Modified) Cramér-Rao Bound** gives the best accuracy that can be attained by **any** unbiased estimator*

$$r(t) = s(t;\tau,\mathbf{c}) + w(t)$$

desired parameter

Vector of unwanted parameter(s): the chips of the ranging code

$$E\left\{\left|\hat{\tau}-\tau\right|^2\right\} \geq \frac{N_0}{E_{\mathbf{c}}\left\{\int_0^{T_0}\left|\frac{\partial s(t;\tau,\mathbf{c})}{\partial \tau}\right|^2 dt\right\}}$$

The MCRB for pseudorange accuracy 1/2

$$\sigma_\tau[\mathrm{m}] \geq cT_c \times \sqrt{\frac{B_L}{2C/N_0}}\left(\frac{1}{2\pi\beta}\right)$$

$B_L = 1/(2T_0)$ *loop bandwidth (we'll see in a while what it means) [Hz]*

C/N_0 *signal-to-noise-ratio per unit bandwdith [dB/Hz]*

cT_c *equivalent chip length [m]*

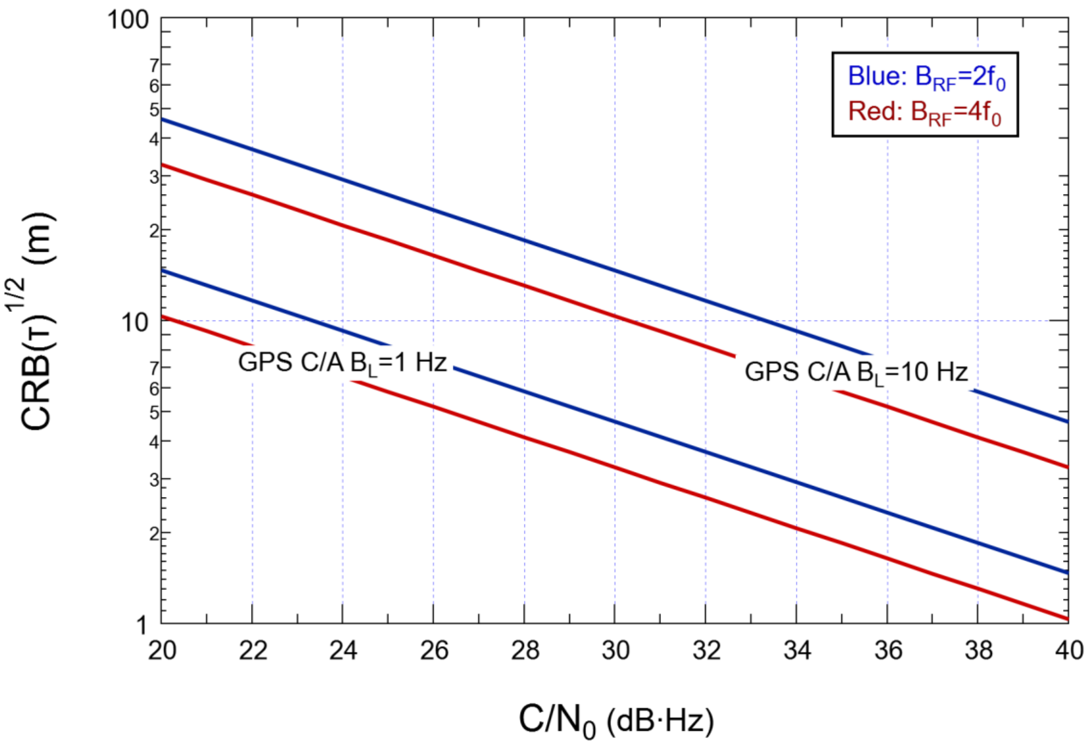
$$\beta^2 = \frac{T_c^2 \int_{-B_RF/2}^{+B_{RF}/2} f^2 |G(f)|^2 \, df}{\int_{-B_{RF}/2}^{+B_{RF}/2} |G(f)|^2 \, df}$$

Normalized Squared Gabor Bandwdith in the receiver (radio) bandwdith B_{RF}

The MCRB for pseudorange accuracy 2/2

$$\sigma_\tau[\mathrm{m}] \geq cT_c \times \sqrt{\frac{B_L}{2C/N_0}}\left(\frac{1}{2\pi\beta}\right)$$

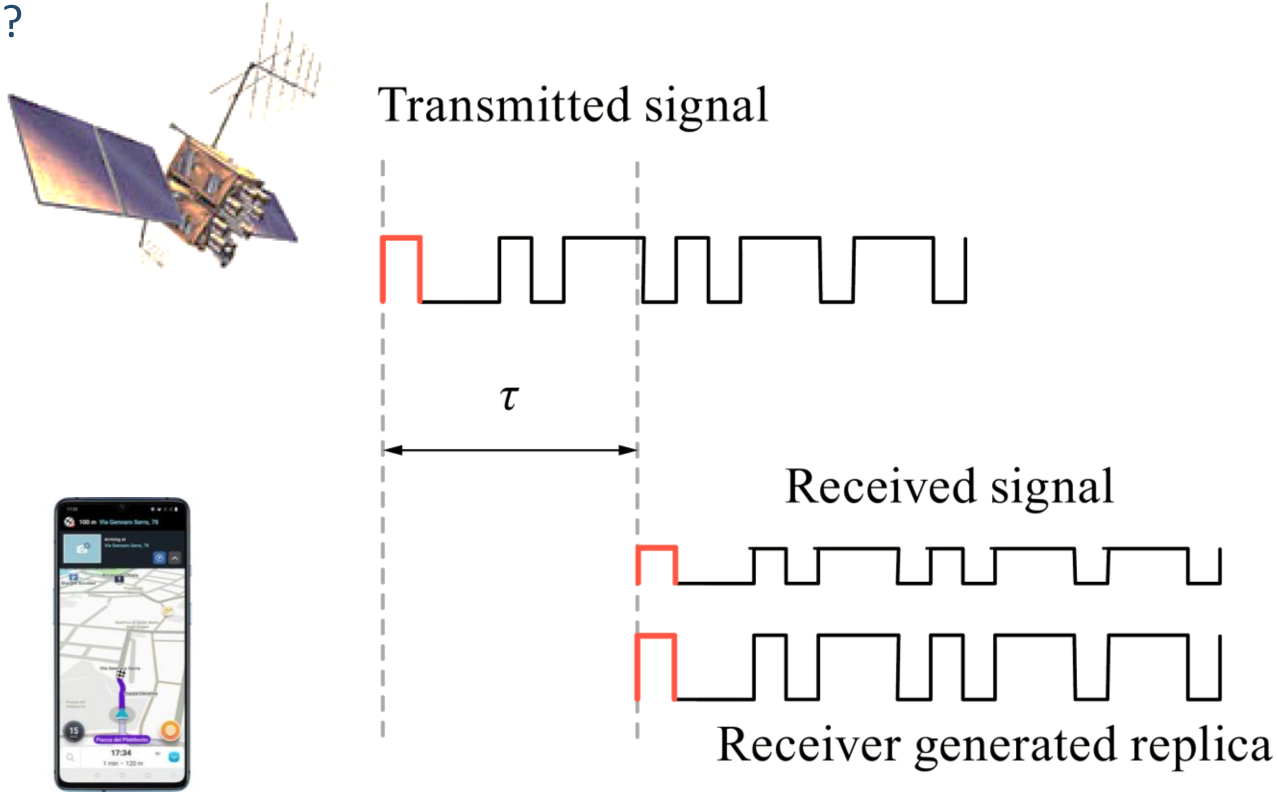
$$\beta^2 = \frac{T_c^2 \int_{-B_R F/2}^{+B_{RF}/2} f^2 |G(f)|^2 \, df}{\int_{-B_{RF}/2}^{+B_{RF}/2} |G(f)|^2 \, df}$$



For usual values of loop bandwidth and SNR, the RMS error due to receiver noise is smaller than 1 m

How far is the Bound? How can we actually estimate the TOA?

- The main idea is simple: implement in the receiver a local ranging code generator, and try to «lock» the local code to the code into the received signal, then measure time based on the local clok of the locked code generator. Is it an *optimal* criterion?

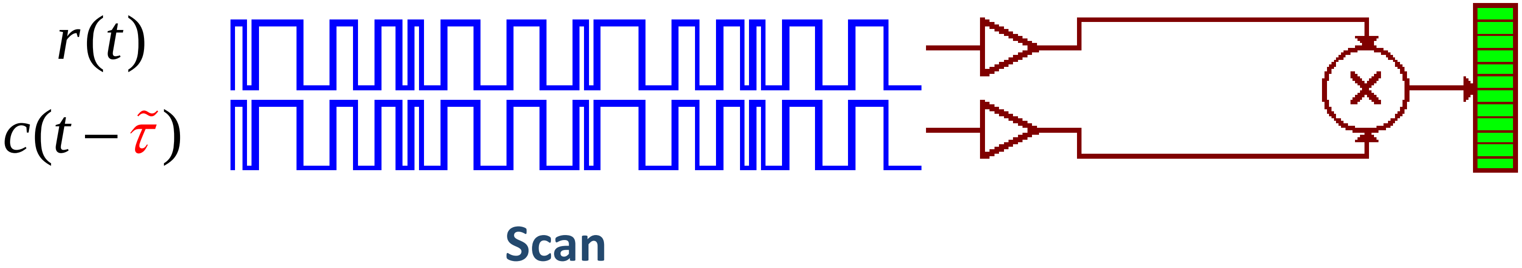


Optimum (Maximum-Likelihood) code acquisition

Simple correlation processing:

$$\hat{\tau} = \arg\max_{\tilde{\tau}} \left\{ \int_0^{T_0} r(t)c(t-\tilde{\tau})dt \right\}$$

Scan the delay values of and find that one that maximizes the cross-correlation between the received signal r *and the local replica code* c



The main issue in TOA estimation

- *GPS C/A code repetition period LT_c : 1 ms (1023 chips)*
- *Typ. required estimation accuracy σ_τ : 0.3 m or 1 ns*
- *$\sigma_\tau / LT_c = 10^{-6}$ very very high relative accuracy – on average, it needs $10^6/2$ tries (scan steps)!*
- *The solution: Need to break-up estimation into COARSE and FINE estimation, or, ACQUISITION and TRACKING*
  - *Tracking comes after acquisition is accomplished and provides the small final required accuracy*
  - *Acquisition provides an initial coarse accuracy of $T_c/2$, thus requiring much less scan steps, then hands over to tracking*

Once acquisition is over...

To maximize $\int_0^{T_0} r(t)c(t-\tilde{\tau})dt$ ***we do:***

$$\frac{d}{d\tilde{\tau}}\int_0^{T_0} r(t)c(t-\tilde{\tau})dt = 0 \Rightarrow \int_0^{T_0} r(t)\frac{d}{d\tilde{\tau}}c(t-\tilde{\tau})dt = 0$$

$$\frac{d}{d\tilde{\tau}}c(t-\tilde{\tau}) \cong -\frac{c(t-\tilde{\tau}+\Delta)-c(t-\tilde{\tau}-\Delta)}{2\Delta} \Rightarrow \int_0^{T_0} r(t)\frac{c(t-\tilde{\tau}+\Delta)-c(t-\tilde{\tau}-\Delta)}{2\Delta}dt = 0$$

i.e., we have to solve

$$\int_0^{T_0} r(t)c(t-\tilde{\tau}+\Delta)dt - \int_0^{T_0} r(t)c(t-\tilde{\tau}-\Delta)dt = 0$$

… fine tracking is started…

$$e(\tilde{\tau}) \triangleq \int_0^{T_0} r(t)c(t-\tilde{\tau}+\Delta)dt - \int_0^{T_0} r(t)c(t-\tilde{\tau}-\Delta)dt$$

Early correlation

Late correlation

Iterative solution

$$\delta[n+1] = \delta[n] - \gamma e(\delta[n])$$

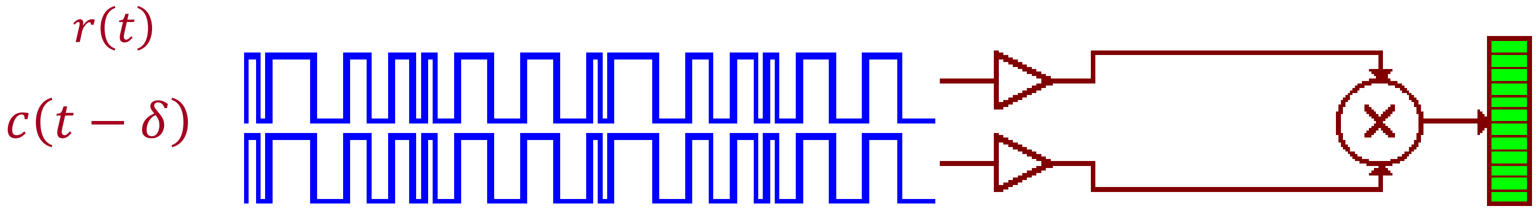
Delay-Lock Loop (DLL) (to be continued)

In reality…

Simple correlation processing:

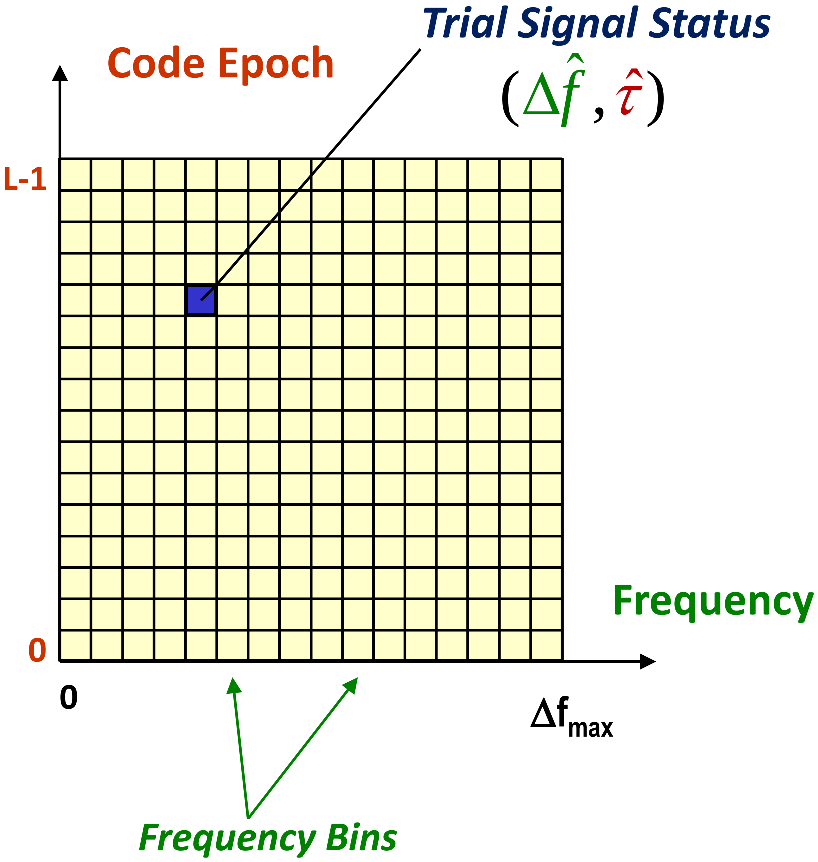
$$\hat{\tau} = \arg\max_{\tilde{\tau}} \left\{ \int_0^{T_0} \underbrace{c(t-\tau)\exp\left[j(2\pi\Delta f t + \theta)\right]}_{pilot\ received\ signal} c(t-\tilde{\tau})dt \right\}$$

It's not so simple because of the presence of the Doppler shift and of an arbitrary phase-shift – processing has to be I/Q and non-coherent (i.e, independent of the carrier phase)



Scan

Initial Code/Frequency Acquisition



Bidimensional Search:

- *The scan has to be carried out both on frequency offset and on code delay (epoch), and it takes time…*
- *It has to be noncoherent (squared modulus of cross-correlation)*
- *It can be sped up using FFTs*

$$(\Delta\hat{f}\,,\hat{\tau}) = \underset{\Delta\tilde{f}\,,\tilde{\tau}}{\arg\max}\left\{\left|\int_0^{T_0} r(t)\exp\left[-j2\pi\Delta\tilde{f}\,t\right]c(t-\tilde{\tau})dt\right|^2\right\}$$

Typical Bi-D Acquisition